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MATHEMATICAL MODELING OF THE HEAT-TRANSFER INTENSITY BY THE METHODS OF THE THEORY OF EXPERIMENT DESIGN

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The conditions that should be realized in the process of continuous mould casting of alloys on the basis of simultaneous account for the factors determining the formation of the near-wall gas interlayer and the heat transfer are elucidated. In this case, forced rejection of the alloy from the wall by a gas blown into the zone of moulding of a casting is provided. In the general case where the gas is blown through a porous wall, near-wall gas cavities merging into a gas film are formed at the working surface of the wall. The problem posed in this investigation is a typical multifactor problem, in which it is quite appropriate to use the method of the theory of experiment design.

One promising new trend in the development of the process of continuous casting is moulding of a casting in a mould without friction on the walls, which can allow one to significantly increase the productivity of the process, improve the quality of castings, and extend the range of cast alloys as compared to those obtained in conventional (metallic or graphite) slip moulds [1]. In this respect, of great interest is the method involving forced rejection of the alloy from the wall by a gas blown in the zone of moulding of a casting [2].

There are a number of technical proposals of methods and apparatuses for solution of this problem [3, 4]. The complexity of the problem requires its step-by-step investigation, and one basic question is a search for the rational conditions of contactless moulding of castings. As yet, there is no unique approach on this point. Thus, in [3] it is noted that the delivery of a gas to the gap between the wall with holes and the melt must be carried out under such conditions (amount, pressure) that the gas does not bubble through the melt [5, 6]. In [4], this process is analyzed on the assumption that a pressure equal to the metallostatic pressure in the melt is produced in the gap between the melt and the wall.

This investigation is devoted to elucidation of the conditions that should be realized in this process on the basis of simultaneous account for the factors determining the formation of the near-wall gas interlayer and the heat transfer. We proceed from the fact that in the general case where a gas is blown through a porous wall [2], near-wall gas cavities merging into a gas film are formed at the working surface of the wall. Moreover, it follows from the calculation results obtained earlier that the smaller the radii of the pores r_h and of the gas cavities R_{cav}, the larger the permissible range of the gas pressure in the interlayer, and the pressure can be maintained at the necessary level in the melt layer [2]. To eliminate the contact of the molten-metal medium with the wall the gas gap must exceed the surface asperities. Accordingly, the value of the minimum gap can be taken to be $(1–1.5)\cdot10^{-5}$ m. Thus, the desirable range of thicknesses of the gas interlayer will roughly be $\xi_{gap} = (1.1–4)\cdot10^{-6}$ m. In this case, the radius of the gas cavities must accordingly be no larger than ξ_{gap} [2, 6], i.e., $R_{cav} = (6–20)\cdot10^{-5}$ m. Based on the above calculations for a 1-m level of the melt mirror (at the metallostatic pressure $P_m = 0.09\cdot10^5$ Pa), the values of R_{cav}, ξ_{gap}, P_c, P_m, and K have been calculated in [2] as functions of the radius of the pores r_h in the wall (for a wide range).

In investigating the characteristics of an apparatus for contactless moulding of a molten-metal medium it is appropriate to use, as the response function [7], the intensity of heat transfer (heat-transfer coefficient) from the solidifying metal to the porous element and then to the solid wall with a cooling system. Under the conditions of a small

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TABLE 1. Coded Values of the Factors and the Response Function at the Indicated Points of the Design

No. of experiment	x_0	x_1	x_2	x_3	x_4	x_5	$G_g \cdot 10^{-3}$	$G_w \cdot 10^{-3}$	$\xi_{gap} \cdot 10^{-6}$	$P_c \cdot 10^5$	$\delta \cdot 10^{-3}$	$K \cdot 10^9$
1	+	–	–	–	–	+	0.3	0.75	15	0.2	6	1.7
2	+	+	–	–	–	–	1.2	0.75	15	0.2	3	2
3	+	–	+	–	–	–	0.3	1.25	15	0.2	3	1.9
4	+	+	+	–	–	+	1.2	1.25	15	0.2	6	1.6
5	+	–	–	+	–	–	0.3	0.75	300	0.2	3	1.1
6	+	+	–	+	–	+	1.2	0.75	300	0.2	6	0.9
7	+	–	+	+	–	+	0.3	1.25	300	0.2	6	0.8
8	+	+	+	+	–	–	1.2	1.25	300	0.2	3	0.8
9	+	–	–	–	+	–	0.3	0.75	15	1	3	1.2
10	+	+	–	–	+	+	1.2	0.75	15	1	6	1.1
11	+	–	+	–	+	+	0.3	1.25	15	1	6	1.0
12	+	+	+	–	+	–	1.2	1.25	15	1	3	0.9
13	+	–	–	+	+	+	0.3	0.75	300	1	6	0.8
14	+	+	–	+	+	–	1.2	0.75	300	1	3	0.7
15	+	–	+	+	+	–	0.3	1.25	300	1	3	0.6
16	+	+	+	+	+	+	1.2	1.25	300	1	6	0.5

gap, we proceed from the assumption that when the gas is delivered through a porous wall near-wall gas cavities merging into a gas film are formed at the working surface of the wall.

It has been established in earlier investigations [1–3] that for a given system of delivery of the gas and a given arrangement of the cooling system (by water, gas, and so on) the heat-transfer coefficient K depends on the following factors: consumption of the working gas G_g, consumption of the cooling material G_w, thickness of the gas interlayer ξ_{gap}, capillary pressure in a gas cavity P_c, and thickness of the porous-element wall δ. Our investigation has been aimed at determining the relationship between the heat-transfer intensity K and the value of the indicated factors.

In accordance with the requirements of the theory of experiment design, the factors must be controlled, measurable, independent, and compatible [8]. This means that having selected the required values of the factors, an experimenter must be able to use any combination of them at points of the design [9] so as to determine the value of the response function in each combination of the values of the factors. In the case considered, the response function, i.e., the intensity of heat transfer in an apparatus of contactless moulding of a molten-metal medium, as well as the factors G_g, G_w, ξ_{gap}, P_c, and δ, conform to the requirements of controllability, independence, measurability, and compatibility [8]. With allowance for the possibilities of the mould tested, the experimental region of the factorial space has the following boundaries:

$$X_1 = G_g = (0.3 - 1.2) \cdot 10^{-3} \text{ kg/sec} ; \quad X_2 = G_w = (0.75 - 1.25) \cdot 10^{-3} \text{ m}^3/\text{sec} ;$$

$$X_3 = \xi_{gap} = (15 - 300) \cdot 10^{-6} \text{ m} ; \quad X_4 = P_c = (0.2 - 1) \cdot 10^5 \text{ Pa} ; \quad X_5 = \delta = (3 - 6) \cdot 10^{-3} \text{ m} .$$

For the convenience of subsequent operations, the independent variables (factors) are usually reduced to dimensionless (coded) variables by relations of the form [7–9]

$$x_j = \frac{X_j - X_{j0}}{J_j} , \tag{1}$$

where $X_{j0} = (X_{max} + X_{min})/2$. The minimum value of each factor in coded values corresponds to –1, while the maximum value corresponds to +1 [9]. The initial level of the factors X_{j0}, the interval of their variation, and their maximum and minimum values are given in Table 1.

Let us consider the features of experiment design under the above-indicated conditions in the case where all the combinations of the limiting values of the factors are realized, i.e., in the case of the full factorial experiment (FFE) of first order [7, 9]. In the case of the full factorial experiment for a linear model, the number of experiments is $N = 2^k$. Since the heat-transfer intensity depends on the five factors, the necessary number of experiments in the full factorial experiment is $2^5 = 32$.

From the results of the full factorial experiment, one can derive the regression equation including all the interactions of the factors to the fourth order inclusive [9]. In this case, interactions of the highest orders are not very significant. The experience obtained in the previous experiments [1–4] suggests that the major part of the interactions are insignificant. This allowed us to restrict ourselves to a simpler description and decrease the volume of the experiment by a factor of two, i.e., to use a fractional factorial experiment, i.e., a half-replicate of a full factorial experiment containing 16 points. The selected half-replicate is characterized by the generating relation $x_5 = x_1x_2x_3x_4$ [8], and the values of the factors at the points of the design correspond to those presented in Table 1. This table also gives the coded values of the factors and the results of determination of the response function at the indicated points of the design. All the measurements were made at points close to those indicated in the table.

From these values, according to formulas of the form

$$b_j = \frac{\sum_{i=1}^{N} K_i x_{ji}}{N}, \tag{2}$$

where x_{ji} is the value of the jth factor in the ith experiment and K_i is the heat-transfer intensity in the ith experiment, we have determined the coefficients of the regression equation.

In the course of the fractional factorial experiment, the estimates of the coefficients are mixed. Thus, for the selected half-replicate determined by the generating relation $x_5 = x_1x_2x_3x_4$ and, consequently, the determining contrast $1 = x_1x_2x_3x_4x_5$, the coefficients determine such mixed (confounded) estimates as

$$b_1 \to \beta_1 + \beta_{2345}, \quad b_2 \to \beta_2 + \beta_{1345}, \quad b_3 \to \beta_3 + \beta_{1245}, \quad b_4 \to \beta_4 + \beta_{1235}, \quad b_5 \to \beta_5 + \beta_{1234}, \quad b_{12} \to \beta_{12} + \beta_{345},$$

$$b_{13} \to \beta_{13} + \beta_{245}, \quad b_{14} \to \beta_{14} + \beta_{235}, \quad b_{15} \to \beta_{15} + \beta_{234}, \quad b_{23} \to \beta_{23} + \beta_{145} \text{ and so on}$$

Here b are the estimates of the coefficients of the regression equation, and β are the true values of the regression coefficients. It is assumed that all the interactions of second, third, and fourth order are insignificant and tend to zero. To statistically estimate the quality of the model obtained, it is necessary to determine the error mean square (reproducibility variance) from the data of m parallel experiments:

$$S^2 = \frac{\sum_{k=1}^{m} (K_{ki} - \overline{K}_i)^2}{m - 1}. \tag{3}$$

Here, $\overline{K}_i$ is the arithmetic mean value of the response function in m parallel experiments. In this work, the error mean square was estimated by the data of five parallel experiments at the center of the design, i.e., at $x_1 = x_2 = x_3 = x_4 = x_5 = 0$.

The confidence interval of the regression coefficients is

$$\Delta b_j = \pm \frac{tS}{\sqrt{N}}. \tag{4}$$

Here, t is the tabular value of the Student criterion for the considered number of degrees of freedom and the selected significance level (usually 0.05). In our case, where the number of degrees of freedom is equal to four and the significance level is $\alpha = 0.05$, the value of the criterion is $t = 2.78$. Comparison of the numerical values of the regression coefficients to the confidence interval allows the conclusion that a coefficient is significant if its absolute value is larger than the confidence interval. The residual variance is

$$S_r^2 = \frac{\sum_{i=1}^{N} (K_i - \hat{K}_i)^2}{N - l}\,, \tag{5}$$

where K_i are the results of the experimental determination of the response function and $\hat{K}_i$ are the data of the calculation of K at the same point from the regression equation.

The calculated Fisher criterion is

$$F_{cal} = \frac{S_r^2}{S^2}\,. \tag{6}$$

If F_{cal} is smaller than the tabular value F_{tab} for the corresponding degrees of freedom of the numerator $f_1 = N - l$ and the denominator $f_2 = m - 1$ at a given significance level, the equation obtained describes the experiment adequately. In the five experiments carried out at the center of the design, we have obtained the following values of K_{i0} (i = 1, 2, ..., 5): K_{10} = 0.97, K_{20} = 0.99, K_{30} = 1.09, K_{40} = 1, and K_{50} = 0.92. Consequently:

$$\overline{K}_0 = 0.994\,, \quad S^2 = \frac{\sum_{i=1}^{5} (K_{i0} - \overline{K}_0)^2}{m - 1} = 38.3 \cdot 10^{-4}\,.$$

Since, in the case considered, t = 2.78, we have $|\Delta b_j| = tS/\sqrt{N} = 2.78 \cdot 6.19 \cdot 10^{-2}/4 = 0.043$.

Comparing the values of the coefficients of the regression equation to the confidence interval Δb_j, it must be borne in mind that all the coefficients at x_i except the coefficient at x_1 are significant, and only the b_{25}, b_{34}, and b_{45} coefficients of the coefficients of pair interaction are significant. Thus, the linear model of the heat-transfer intensity in an apparatus for contactless moulding of molten-metal media has the form

$$\hat{K} = b_0 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_{25} x_2 x_5 + b_{34} x_3 x_4 + b_{45} x_4 x_5\,.$$

Substituting coefficients b, we obtain

$$\hat{K} = 1.1 - 0.0875 x_2 - 0.325 x_3 - 0.25 x_4 - 0.05 x_5 - 0.0563 x_2 x_5 + 0.125 x_3 x_4 + 0.05 x_4 x_5\,. \tag{7}$$

In order that the relation obtained can be used in practice, it is necessary to bring it to the full scale by formula (1). After rearrangements, the above dependence takes the form

$$\hat{K} = 2.09 + 0.396 G_w - 0.003 \xi_{gap} - 1.345 P_c + 0.067 \delta - 0.15 G_w \delta + 0.002 \xi_{gap} P_c + 0.083 P_c \delta\,. \tag{8}$$

The residual variance is

$$S_r^2 = \frac{\sum_{i=1}^{N} (K_i - \hat{K}_i)^2}{N - l}\,, \quad N = 16, \quad l = 8.$$

Substitution of the results of the algebraic calculations into the formula for the residual variance gives S_r^2 = 0.0166. Then, according to the Fisher criterion:

$$F_{cal} = \frac{S_r^2}{S^2} = 4.33\,; \quad F_{tab} = F_{0.05}\,(N - l\,;\, m - 1) = F_{0.05}\,(8\,;\,4) = 6.04\,.$$

Since $F_{cal} < F_{tab}$, the model obtained is adequate to the experimental data. To verify the quality of the model obtained and separate the nonmixed influence of individual parameters and the effects of interaction, we added 16 more experiments necessary for realization of the design of the full factorial experiment to the experiments of a half-replicate. The confidence interval of the regression coefficients is

$$|\Delta b_j| = \frac{tS}{\sqrt{N}} = 0.03 \,.$$

When the coefficients of the regression equation, obtained in the previous case, are compared to the confidence interval Δb_j, it is seen that all the coefficients b_j are significant, the coefficients b_{12}, b_{34}, and b_{45} of the coefficients of pair interaction are significant, and only the coefficient b_{1235} of the coefficients characterizing the interactions of third order is significant. The coefficient b_{12345} is also significant. All the other coefficients are insignificant. Thus, the linear model of the heat-transfer intensity has the form

$$\hat{K} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{34}x_3x_4 + b_{45}x_4x_5 + b_{1235}x_1x_2x_3x_5 + b_{12345}x_1x_2x_3x_4x_5 \,.$$

After substitution of the obtained values of the coefficients b, the dependence takes the form

$$\hat{K} = 1.17 - 0.016x_1 - 0.117x_2 - 0.305x_3 - 0.298x_4 - 0.07x_5 - 0.036x_1x_2 +$$

$$+ 0.102x_3x_4 + 0.036x_4x_5 + 0.048x_1x_2x_3x_5 - 0.07x_1x_2x_3x_4x_5 \,. \quad (9)$$

Statistical estimation of the quality of the model obtained is made in the same way as for the model obtained in the fractional factorial experiment:

$$S^2 = 38.3 \cdot 10^{-4} \,, \quad S_r^2 = \frac{\sum_{i=1}^{N} (K_i - \hat{K}_i)^2}{N - l} \,, \quad N = 32 \,, \quad l = 11 \,; \quad S_r^2 = 0.008 \,.$$

Then, according to the Fisher criterion:

$$F_{cal} = \frac{S_r^2}{S^2} = \frac{0.008}{38.3 \cdot 10^{-4}} = 2.09 \,, \quad F_{tab} = F_{0.05}\,(N - l \,; m - 1) = F_{0.05}\,(21 \,; 4) = 5.8 \,.$$

If $F_{cal} < F_{tab}$, the model obtained describes the experimental data adequately.

To use the equation obtained in practice it is necessary to bring it to the full scale by formula (1). Then relation (9) will take the form

$$\hat{K} = 6.205 - 4.326G_g - 3.609G_w - 0.03\xi_{gap} - 5.163P_c - 0.833\delta + 4.19G_gG_w + 0.03G_g\,\xi_{gap} + 5.16G_gP_c +$$

$$+ G_g\delta + 0.03G_w\,\xi_{gap} + 3.868G_wP_c + 0.75G_w\delta + 0.03P_c\,\xi_{gap} + 0.005\xi_{gap}\delta + 0.92P_c\delta - 0.03G_g\,\xi_{gap}G_w -$$

$$- 5.16G_gP_cG_w - 0.001G_g\delta G_w - 0.033G_g\,\xi_{gap}P_c - 0.004G_w\,\xi_{gap}\delta - 0.858P_{ê}G_w\delta - 0.006P_c\xi_{gap}\delta +$$

$$+ 0.033G_wG_g\,\xi_{gap}P_c + 0.006G_wG_g\,\xi_{gap}\delta + 1.146G_gG_wP_c\delta + 0.007G_g\,\xi_{gap}P_c\delta + 0.006G_wP_c\delta\,\xi_{gap} - 0.007G_{â}P_{ê}\delta\,\xi_{ç}G_{ã} \,.$$

It should be noted that the mean value of the function at the center of the design (according to the parallel experiments) $\overline{K} = 0.994$ differs markedly from the value $K_0 = b_0 = 1.17$ obtained with the help of the linear model. Despite the fairly high accuracy of the obtained linear model of the heat-transfer intensity in an apparatus for contactless moulding of a molten-metal medium, the regularities investigated can be expressed in the form of a power function [9]:

$$\hat{K} = b_0 (G_g)^{b1} (G_w)^{b2} \xi_{gap}^{b3} P_c^{b4} \delta^{b5} . \tag{10}$$

Having taken the logarithm of the above dependence, we obtain a linear model relative to the new factors Z_i = ln X_i without interactions

$$Y = \ln \hat{K} = \ln b_0 + b_1 \ln (G_g) + b_2 \ln (G_w) + b_3 \ln (\xi_{gap}) + b_4 \ln (P_c) + b_5 \ln \delta$$

or

$$Y = \alpha_0 + b_1 Z_1 + b_2 Z_2 + b_3 Z_3 + b_4 Z_4 + b_5 Z_5 .$$

Having conducted N experiments with simultaneous variation of our five factors, we write Eq. (10) in matrix form:

$$\mathbf{Y} = \mathbf{Z} \cdot \mathbf{B} , \tag{11}$$

where

$$\mathbf{Z} = \begin{bmatrix} z_0^1 & z_1^1 & z_2^1 & z_3^1 & z_4^1 & z_5^1 \\ & \dots & \dots & \dots & \dots & \\ & \dots & \dots & \dots & \dots & \\ z_0^i & z_1^i & z_2^i & z_3^i & z_4^i & z_5^i \\ & \dots & \dots & \dots & \dots & \\ & \dots & \dots & \dots & \dots & \\ z_0^N & z_1^N & z_2^N & z_3^N & z_4^N & z_5^N \end{bmatrix} = \begin{bmatrix} 1 & \ln (X_1^1) & \ln (X_2^1) & \ln (X_3^1) & \ln (X_4^1) & \ln (X_5^1) \\ & \dots & \dots & \dots & \dots & \\ & \dots & \dots & \dots & \dots & \\ 1 & \ln (X_1^i) & \ln (X_2^i) & \ln (X_3^i) & \ln (X_4^i) & \ln (X_5^i) \\ & \dots & \dots & \dots & \dots & \\ & \dots & \dots & \dots & \dots & \\ 1 & \ln (X_1^N) & \ln (X_2^N) & \ln (X_3^N) & \ln (X_4^N) & \ln (X_5^N) \end{bmatrix} ,$$

$$\mathbf{B} = \begin{bmatrix} \alpha_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix} , \quad \mathbf{Y} = \begin{bmatrix} \ln K_1 \\ \dots \\ \dots \\ \ln (K_i) \\ \dots \\ \dots \\ \ln (K_N) \end{bmatrix} .$$

According to the theory of experiment design, the least-squares method is used for numerical estimation of the coefficients of the polynomial model describing the behavior of the system investigated. To determine the regression coefficients by this method, it is necessary to minimize the sum of the squares of deviations:

$$\sum_{i=1}^{N} (\ln K_i - \alpha_0 z_0^i - b_1 z_1^i - b_2 z_2^i - \dots - b_5 z_5^i)^2 .$$

Setting the partial derivatives of this square form with respect to variables α_0, b_1, b_2, b_3, b_4, and b_5 equal to zero, we obtain the system of so-called normal equations

$$\alpha_0 \sum_{i=1}^{N} (z_0^i)^2 + b_1 \sum_{i=1}^{N} z_0^i z_1^i + \dots + b_5 \sum_{i=1}^{N} z_0^i z_5^i = \sum_{i=1}^{N} z_0^i \ln K_i ;$$

$$\alpha_0 \sum_{i=1}^{N} z_0^i z_1^i + b_1 \sum_{i=1}^{N} (z_1^i)^2 + ... + b_5 \sum_{i=1}^{N} z_1^i z_5^i = \sum_{i=1}^{N} z_1^i \ln K_i \ ;$$

$$... \quad ... \quad ... \quad ... \quad ...$$

$$\alpha_0 \sum_{i=1}^{N} z_0^i z_5^i + b_1 \sum_{i=1}^{N} z_5^i z_1^i + ... + b_5 \sum_{i=1}^{N} (z_5^i)^2 = \sum_{i=1}^{N} z_5^i \ln K_i \ .$$

To find the regression coefficients of interest to us, it is necessary to solve the system of normal equations relative to unknown quantities α_0, b_1, b_2, b_3, b_4, and b_5.

In matrix form, the system of normal equations has the form

$$\mathbf{Z}^t \cdot \mathbf{Z} \cdot \mathbf{B} = \mathbf{Z}^t \cdot \mathbf{Y} \ . \tag{12}$$

On condition that $\mathbf{Z}^t \cdot \mathbf{Z}$ is the nondegenerate matrix (where the superscript t means transposition), we find the matrix $(\mathbf{Z}^t \cdot \mathbf{Z})^{-1}$ reverse to the matrix $\mathbf{Z}^t \cdot \mathbf{Z}$. Having multiplied from the left both sides of the matrix equation (12) by $(\mathbf{Z}^t \cdot \mathbf{Z})^{-1}$, we obtain $(\mathbf{Z}^t \cdot \mathbf{Z})^{-1} \cdot (\mathbf{Z}^t \cdot \mathbf{Z}) \cdot \mathbf{B} = \mathbf{B} = (\mathbf{Z}^t \cdot \mathbf{Z})^{-1} \cdot \mathbf{Z}^t \cdot \mathbf{Y}$.

The regression coefficients are determined by the expressions

$$\alpha_0 = \sum_{k=0}^{5} c_{0k} \sum_{j=1}^{N} z_0^j \ln K_j \ ; \quad b_i = \sum_{k=0}^{5} c_{ik} \sum_{j=1}^{N} z_0^j \ln K_j \ ,$$

where c_{ik} is the element of the $(\mathbf{Z}^t \cdot \mathbf{Z})^{-1}$ matrix at the intersection of the ith line and the kth column of this matrix.

The foregoing allows the conclusion that the coefficients of the linear model obtained after taking the logarithm can be determined, as earlier, from a multifactorial experiment (full factorial experiment or fractional factorial experiment). Since in this case the linear design contains only limiting values (± 1), the design matrix of the fractional factorial experiment will be the same as earlier (only the center of the design shifts), and the initial data can be used to calculate the regression coefficients. To obtain the error mean square, we used the data of m = 5 experiments at the center of the design K_{i0}, where i = 1, ..., 5, and made the following calculations:

$$Y_{10} = \ln K_{10} = -0.03 \ ; \quad Y_{20} = \ln K_{20} = -0.01 \ ; \quad Y_{30} = \ln K_{30} = 0.086 \ ; \quad Y_{40} = \ln K_{40} = 0 \ ;$$

$$Y_{50} = \ln K_{50} = -0.083 \ ; \quad \overline{Y}_0 = \frac{1}{m} \sum_{i=1}^{m} \ln K_{i0} = \frac{1}{5} \sum_{i=1}^{5} \ln K_{i0} = -0.0074 \ .$$

Then the sought variance is equal to

$$S^2 = \frac{1}{m-1} \sum_{i=1}^{m} (Y_{i0} - \overline{Y}_0)^2 = 3.75 \cdot 10^{-3} \ .$$

The values of **Z** and **Y** obtained from the experimental data within the framework of the theory of experiment design allow one to determine the vector **B**:

$$\mathbf{B} = \begin{bmatrix} \alpha_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix} = \begin{bmatrix} 0.734 \\ -0.006 \\ -0.388 \\ -0.198 \\ -0.269 \\ -0.103 \end{bmatrix}, \quad b_0 = \exp(a_0) = \exp(0.734) = 2.083 \ .$$

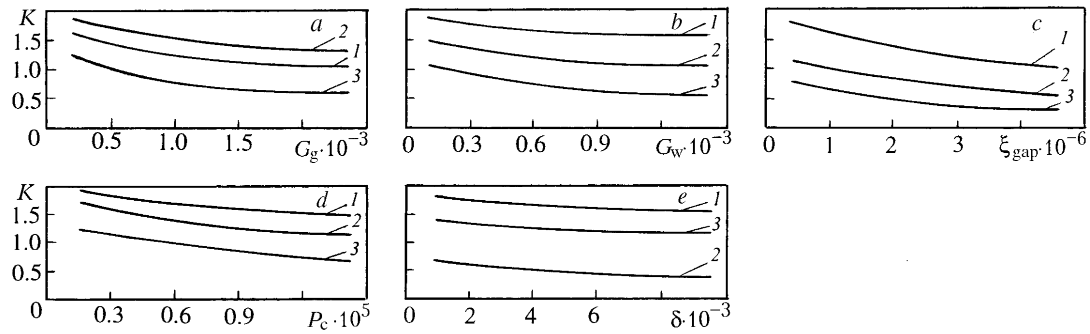

Fig. 1. Change in the heat-transfer intensity as a function of the working consumption of the gas (a), consumption of the water for $\delta = 3 \cdot 10^{-3}$ m (b), thickness of the gas interlayer (c), capillary pressure in a gas cavity (d), and thickness of the porous-element wall (e): a: 1) $G_w = 8 \cdot 10^{-4}$, $\xi_{gap} = 1.5 \cdot 10^{-6}$, $P_c = 0.3 \cdot 10^5$, and $\delta = 3 \cdot 10^{-3}$; 2) $7.5 \cdot 10^{-4}$, $1.5 \cdot 10^{-6}$, $0.2 \cdot 10^5$, and $6 \cdot 10^{-3}$; 3) $1 \cdot 10^{-3}$, $4 \cdot 10^{-6}$, $1 \cdot 10^5$ and $5 \cdot 10^{-3}$; b: 1) $G_g = 0.6 \cdot 10^{-3}$, $\xi_{gap} = 1.5 \cdot 10^{-6}$, and $P_c = 0.3 \cdot 10^5$; 2) $0.3 \cdot 10^{-3}$, $3 \cdot 10^{-4}$, and $0.2 \cdot 10^5$; 3) $1.2 \cdot 10^{-3}$, $4 \cdot 10^{-5}$, and $1 \cdot 10^5$; c: 1) $G_w = 8 \cdot 10^{-4}$, $G_g = 0.6 \cdot 10^{-3}$, $P_c = 0.3 \cdot 10^5$, and $\delta = 3 \cdot 10^{-3}$; 2) $12.5 \cdot 10^{-4}$, $0.3 \cdot 10^{-3}$, $1 \cdot 10^5$, and $3 \cdot 10^{-3}$; 3) $1 \cdot 10^{-3}$, $0.3 \cdot 10^{-3}$, $1 \cdot 10^5$, and $4 \cdot 10^{-3}$; d: 1) $G_w = 8 \cdot 10^{-4}$, $\xi_{gap} = 1.5 \cdot 10^{-5}$, $G_g = 0.6 \cdot 10^{-3}$, and $\delta = 3 \cdot 10^{-3}$; 2) $7.5 \cdot 10^{-4}$, $1.5 \cdot 10^{-5}$, $1.2 \cdot 10^{-3}$, and $3 \cdot 10^{-3}$; 3) $1 \cdot 10^{-3}$, $1.5 \cdot 10^{-5}$, $1.2 \cdot 10^{-3}$, and $4 \cdot 10^{-3}$; e: 1) $G_w = 8 \cdot 10^{-4}$, $\xi_{gap} = 1.5 \cdot 10^{-5}$, $P_c = 0.3 \cdot 10^5$, and $G_g = 0.6 \cdot 10^{-3}$; 2) $12.5 \cdot 10^{-4}$, $3 \cdot 10^{-4}$, $1 \cdot 10^5$, and $1.2 \cdot 10^{-3}$; 3) $1 \cdot 10^{-3}$, $3 \cdot 10^{-4}$, $1 \cdot 10^5$, and $1.2 \cdot 10^{-3}$.

Finally the regression equation has the form

$$\hat{K} = 2.083\,(G_g)^{-0.066}\,(G_w)^{-0.388}\,\xi_{gap}^{-0.198}\,(P_c)^{-0.269}\,\delta^{-0.103}\,. \tag{13}$$

The adequacy of the equation obtained was verified by the Fisher criterion:

$$S_r^2 = \frac{1}{N - l} \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2\,, \quad N = 16\,, \quad l = 6\,,$$

$$S_r^2 = 0.011\,, \quad F_{cal} = \frac{S_r^2}{S^2} = \frac{0.011}{3.75 \cdot 10^{-3}} = 2.9\,, \quad F_t = F_{0.05}\,(10\,;\,4) = 5.96\,.$$

The analogous processing of the results of the experiments according to the design of the full factorial experiment with the additional 16 experiments (in the case of separation of the nonmixed influence of individual parameters by a half-replicate) gives the following mathematical model of the heat-transfer intensity:

$$\hat{K} = 1.784\,(G_g)^{-0.139}\,(G_w)^{-0.344}\,\xi_{gap}^{-0.159}\,(P_c)^{-0.272}\,\delta^{-0.074}\,. \tag{14}$$

The adequacy of the equation obtained to the experimental data was verified by the Fisher criterion. In this case, $F_{cal} = 5.3$, i.e., $F_{cal} < F_{tab} = 5.7$. Consequently, the equation obtained describes the experimental data adequately. A comparison of relations (13) and (14) and the results of estimation of the adequacy of these relations to the experimental data show that the fractional factorial experiment as well as the full factorial experiment allow one to obtain sufficiently accurate mathematical models that differ only slightly. To verify the possibility of using an even smaller number of experiments (the model contains only six members; therefore, even a half-replicate of 16 experiments is redundant), we processed the experiments corresponding to one quarter-replicate. We obtained the following exponential model:

$$\hat{K} = 2.428\,(G_g)^{-0.255}\,(G_w)^{-0.268}\,\xi_{gap}^{-0.164}\,(P_c)^{-0.293}\,\delta^{-0.341}\,.$$

Statistical verification of the model has shown that $S^2 = 3.75 \cdot 10^{-3}$, $F_{cal} = 2.8$, $F_{tab} = 6.94$, and $F_{cal} < F_{tab}$. Consequently, the model obtained describes the experiment adequately.

Thus, the use of the theory of fractional factorial experiment allows one to significantly decrease the number of experiments and obtain an adequate mathematical model of the heat-transfer intensity in the characteristic range of variation of the parameters determining the working capacity of a mould for contactless moulding of a molten-metal medium. The calculation results are presented in Fig. 1.

The use of the theory of experiment design (full factorial experiment or fractional factorial experiment) allows one to significantly decrease the number of adjusting experiments and obtain an adequate mathematical model of the heat-transfer intensity in the characteristic range of variation of the parameters determining the working capacity of an apparatus for contactless moulding of alloys. It is recommended that the results obtained be taken into account in subsequent investigations, especially in analysis of conditions providing the formation and maintenance of a thin gas interlayer between the molten-metal medium and the porous wall of a compact stationary apparatus in the zone of moulding of a casting.

NOTATION

R_{cav}, radius of the gas cavity in the near-wall layer, m; P_c, capillary pressure in the gas cavity, Pa; r_h, radius of the outlet hole in the porous wall, m; P_m, metallostatic pressure at the level of 1 m, Pa; K, heat-transfer coefficient, $W/(m^2 \cdot K)$; ξ_{gap}, value of the gas interlayer (gap), m; G_g, consumption of the gas rejecting the melt, kg/m^3; G_w, consumption of the water necessary for cooling of the working wall, kg/m^3; δ, thickness of the porous-element wall, m; X_j (j = 0, 1, 2, 3, 4, 5), factors; X_{max} and X_{min}, maximum and minimum values of the factor; X_{j0}, initial level of the jth factor; J_j, interval of variation of the jth factor; x_j, dimensionless value of the jth factor; N, number of experiments; k, number of factors; m, number of parallel experiments; b_j, coefficient of the regression equation; Δb_j, confidence interval of the regression coefficients; t, tabular value of the Student criterion; S_r, residual variance; α, significance level; K_i, results of experimental determination of the response function; $\hat{K}_i$, values of the quantity K determined from the regression equation; l, number of significant coefficients in the regression equation; F_{cal}, calculated Fisher criterion; F_{tab}, tabular value of the Fisher criterion; f, number of degrees of freedom.

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